

Cover page

Title: Finite Difference Modeling and Simulation of Idealized Gear Vibrations

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ABSTRACT

A physics based first principles approach is adopted to model and simulate vibration signatures from an idealized gear such as a high aspect ratio spur gear. The governing equations are solved using a finite-difference approach. The velocity-stress formulation of linear elasticity as used in earthquake signature modeling has been used in the present study, except that in the present case, the velocity-stress system is solved in generalized curvilinear coordinates and that the system is essentially dynamic. A fully characteristic set of boundary conditions based on the theory of hyperbolic systems derived earlier by the author is used in this study. The vibration signatures are thus directly obtained in the time domain. A second-order accurate in time and space time-staggered leap-frog scheme is used to integrate the time-dependent partial differential equations. Idealized signatures, normal as well as damage vibration signatures, are obtained and compared; normal signature is taken to be the one correspondent with the impulsive rotation of the gear without any impact loading. Damage signatures correspond to a case where the rigidity of one of the gear teeth is locally reduced. It is observed that significant deviations from the normal signature occur in amplitude and phase due to this damage. Using this approach, baseline or reference signatures can be obtained for any structural subsystem which can be used to calibrate and validate various damage detection algorithms for such systems.

INTRODUCTION

Modeling and simulation of free and forced structural vibrations is an essential element of an overall health monitoring capability for any structural system such as a rotorcraft or any aerospace vehicle. In the present paper, a first principles finite-difference approach is adopted in modeling a structural subsystem such as a mechanical gear by solving elastodynamic equations in generalized curvilinear coordinates. Any other structural subsystem can similarly be modeled; here, the case of a gear is considered just as an example. Such a capability to generate a dynamic structural response has a wide applicability in a variety of structural health monitoring systems. Not only does this capability serve as a tool for understanding the dynamic behavior of a structural system and hence its improved design, but it also serves as a means by which a sufficiently large space of normal and damage solutions can be generated that can be used by a variety of machine learning algorithms to detect anomalous dynamic structural behavior of the system or to achieve a multi-function design optimization of the given structural

system. This capability will also aid in defining an optimal sensor placement configuration over structural subsystems for health monitoring, by identifying areas of local maxima of mechanical or thermal stress or loading. Such a capability to generate vibration response from a subsystem will also be useful in the area of vibration energy harvesting. Also, the methodology can be used to track stress wave propagation in a structural system which is useful in the health monitoring of such a system.

RESULTS

Normal vibrations of an idealized multi-teeth steel gear are simulated by impulsively rotating the gear. This throws the gear into free vibrations about an equilibrium state that would be attained by it, if it had been set into rotation gradually from an initial state of rest. After about two rotations, the gear attains this steady state. Impact vibrations of the gear simulated by impacting an individual gear tooth, once per gear rotation, after the gear has completed two rotations from an initial state of rest, will be presented later. But, the results presented here correspond to two rotations of the gear after impulsive start, with a tooth damage.

The simulation considered corresponds to all the gear teeth except one, labeled tooth #1, having uniform material properties as those of industrial steel. The shear modulus or the rigidity of tooth #1 is decreased in a certain fashion over the region shown in Fig. 1. This is just to mimic a damage state that would yield distinctly different vibration signatures from this particular tooth from those from the rest of the gear teeth. The gear is impulsively rotated at 6,000 rpm. The elastodynamic partial differential equations (pde), three for the velocity vector and six for the symmetric stress tensor, are integrated in time (Refs. 1 and 2), using fully characteristic boundary conditions (see Refs. 3 and 4). An attractive element of the characteristic boundary condition approach is that the artificial wave attenuation and wave reflection problems associated with the traditional boundary condition approach are entirely eliminated. The velocity-stress formulation of the elastodynamic pde has been used in geophysics to predict reference earthquake signatures (Refs. 5 –7). The integration is carried out until the end of second rotation of the gear, when the vibrations have essentially died out and the equilibrium stress state is achieved. Thus the steady state solution is obtained for radial, tangential and shear stress distribution all over the gear in constant rotation at 6,000 rpm. No grid independence study has been carried out presently. But, having conducted the grid independence study using the present methodology earlier (Ref. 1), the accuracy of the results presented here should be adequate in the present context. Also, as shown in Ref. 1, a small measure of Coriolis effect may be present even at 6,000 rpm.

The results from the simulation are compared side by side in terms of the time evolution of radial, tangential and shear stresses at selected locations on two selected gear teeth, tooth #1 and tooth #10, as shown in Figures 2 – 4 below. Also shown is the steady state distribution of radial stress, all over the gear, in Fig. 4 below. The radial and tangential directions in the present case correspond to the two orthogonal generalized curvilinear coordinates, as shown in Fig. 1.

A comparison of signatures from the damaged tooth #1 and a normal tooth #10 demonstrates that they vary significantly from each other. Also, certain insights can be drawn from simulations such as the present one that would aid in better design technologies of such systems. A detailed study of the results of such simulations as also those corresponding to an impact tooth loading will be carried out and presented in the near future.

For damage detection on a given structural subsystem such as a gear system, various machine learning methods are being trained and tested currently

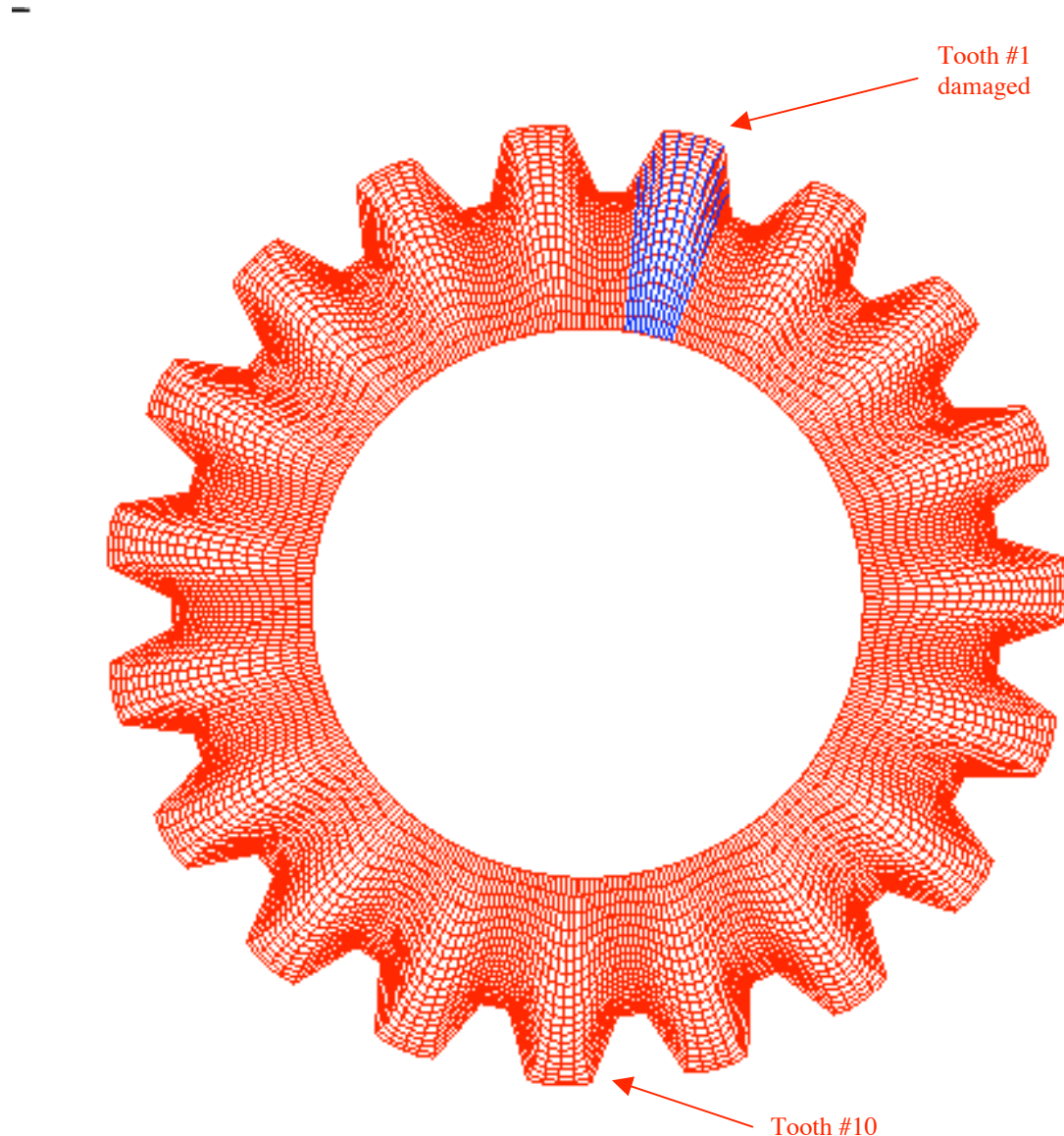


Figure 1. A gear grid cross-section showing a tooth region with reduced rigidity

(Refs. 8 and 9) on the vibration and steady state stress data provided through the present study to demonstrate the viability of the structural monitoring capability through modeling and simulation of a given structural subsystem.

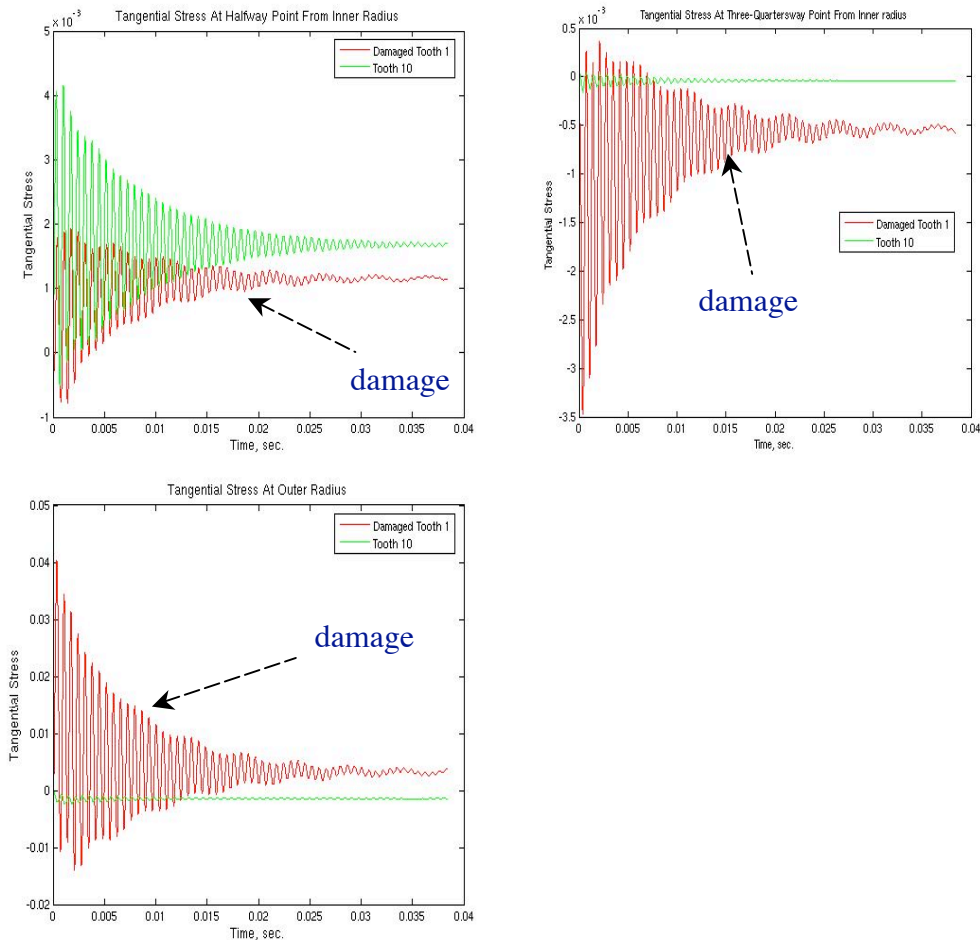
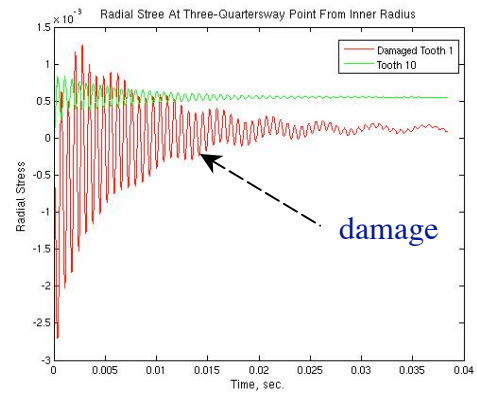
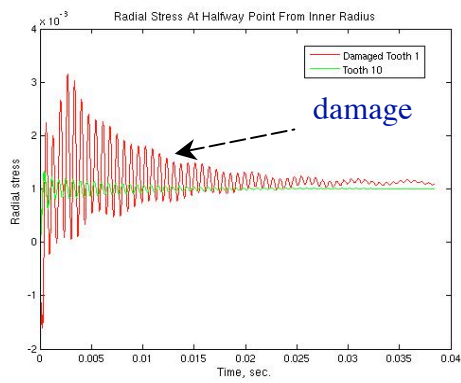


Figure 2. Comparison between normal and damage tangential stress signatures: time evolution (over two rotations) at various locations between the inner radius and outer radius, at corresponding locations below tooth #1 and tooth #10



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Figure 3. Comparison between normal and damage radial stress signatures: time evolution (over two rotations) at various locations between the inner radius and outer radius, at corresponding locations below tooth #1 and tooth #10

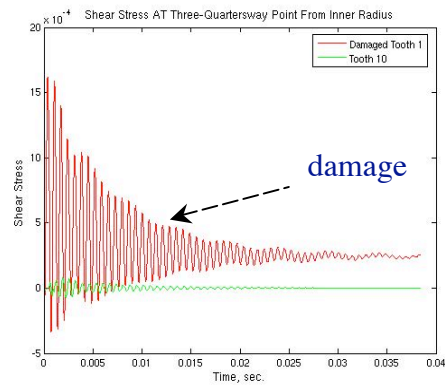
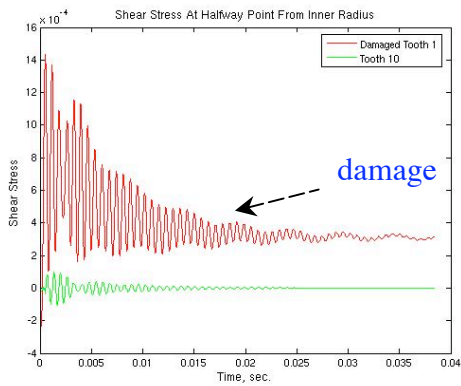


Figure 4. Comparison between normal and damage shear stress signatures: time evolution (over two rotations) at various locations between the inner radius and outer radius, at corresponding locations below tooth #1 and tooth #10

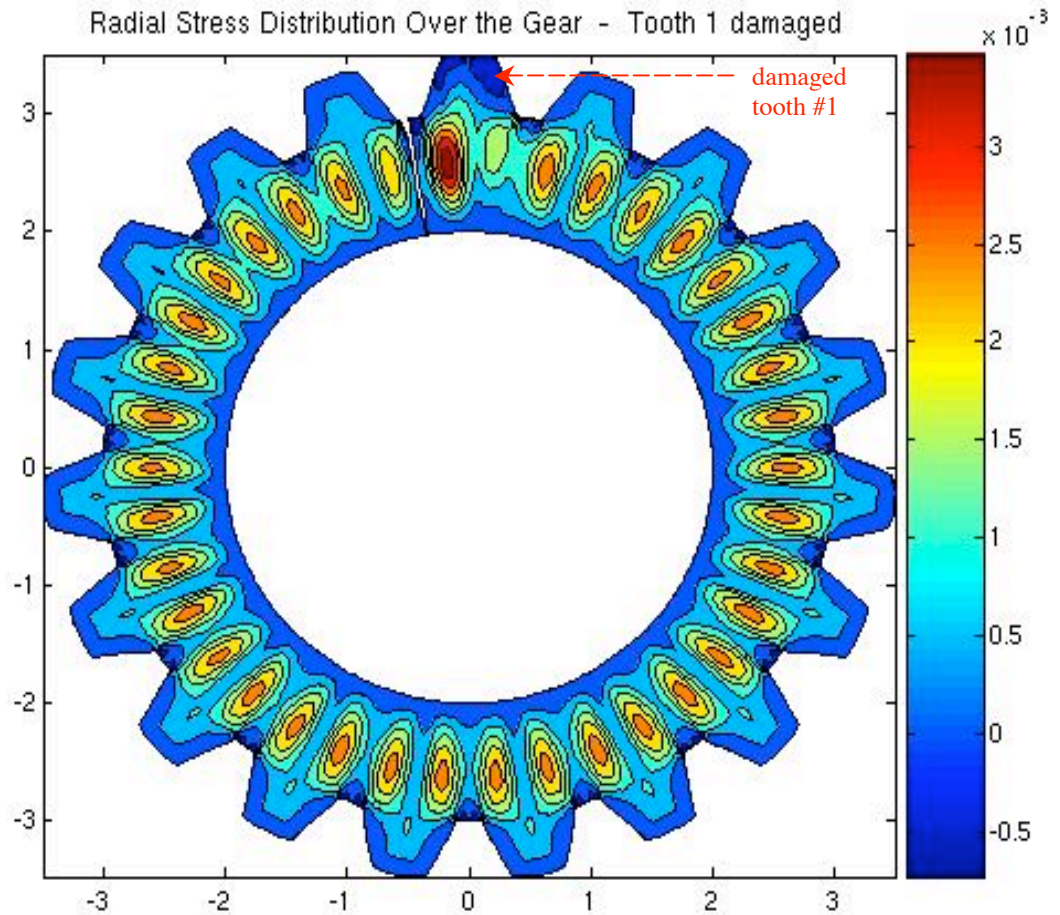


Figure 5. Steady state radial stress distribution at the end of second rotation after impulsive start; tooth #1 is damaged.

CONCLUDING REMARKS AND FUTURE WORK

A computational modeling and simulation methodology, based on first principles, has been developed that can predict various damage states of a given structural system of arbitrary geometry. These simulations can be used in conjunction with damage detection algorithms such as wavelets and machine learning methods for isolation, diagnosis and prognosis of system damage states. Such structural damage, amongst others, can be caused by unwanted material property variation due to thermal effects or manufacturing faults, sudden impact and repetitive loading, during the system operation. A wide variety of these physics-based simulations will be carried out to compute various forms of normal and damage signatures. These signatures will then be used as reference signatures to calibrate and validate various damage detection algorithms.

ACKNOWLEDGEMENTS

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REFERENCES

1. Kaul, U. K., "FiDDLE: A Computer Code for Finite Difference Development of Linear Elasticity in Generalized Curvilinear Coordinates," NASA/TM-2005-213450, January 2005
2. Kaul, U. K., "New Boundary Constraints for Elliptic Systems Used in Grid Generation Problems, Journal of Computational Physics," Vol. 189,2003, pp 476-492
3. Kaul, U. K., "Structural Vibration Signature Modeling," Industry Day, NASA Ames Research Center, 2004
4. Kaul, U. K., "Finite Difference Modeling and Simulation of Reference Vibration Signatures," Internal NASA documents, March 2004
5. Madariaga, R., "Dynamics of an expanding circular fault," Bulletin of the Seismological Society of America, vol. 65, 1976, pp 163-182
6. Virieux, J., "SH-wave propagation in heterogeneous media: velocity-stress finite-difference method", J. Geophysics, vol. 49, no. 11, November 1984, pp 1933-1957
7. Graves, R., "SH-wave propagation in heterogeneous media: velocity-stress finite-difference method", J. Geophysics, vol. 49, no. 11, November 1984, pp 1933-1957
8. Kaul, U. K. and N. C. Oza, "Machine learning for optimal sensor placement in a rotating gear," SAE World Aerospace Conference, Paper 05WAC-121, October 3 - 6, 2005, Dallas, Texas
9. Oza, N. C. and U. K. Kaul, "Machine learning for fault detection in a rotating gear," SAE World Aerospace Conference, Paper 05WAC-118, October 3 - 6, 2005, Dallas, Texas